Post-quantum key exchange for the TLS protocol from the ring learning with errors problem

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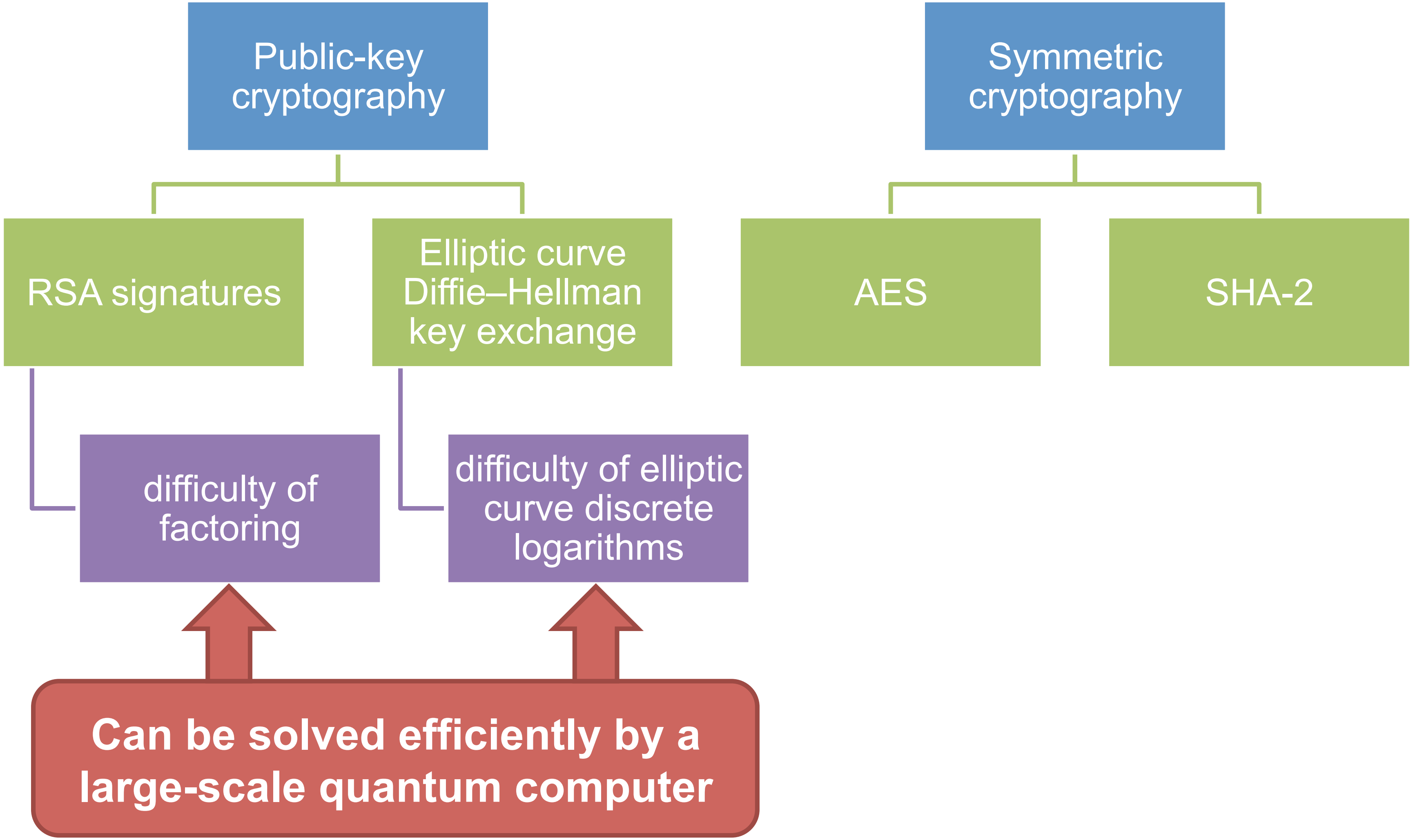
Background

Contemporary cryptography

TLS-ECDHE-RSA-AES128-GCM-SHA256



Public-key cryptography

Symmetric cryptography

RSA signatures

Elliptic curve Diffie–Hellman key exchange

AES

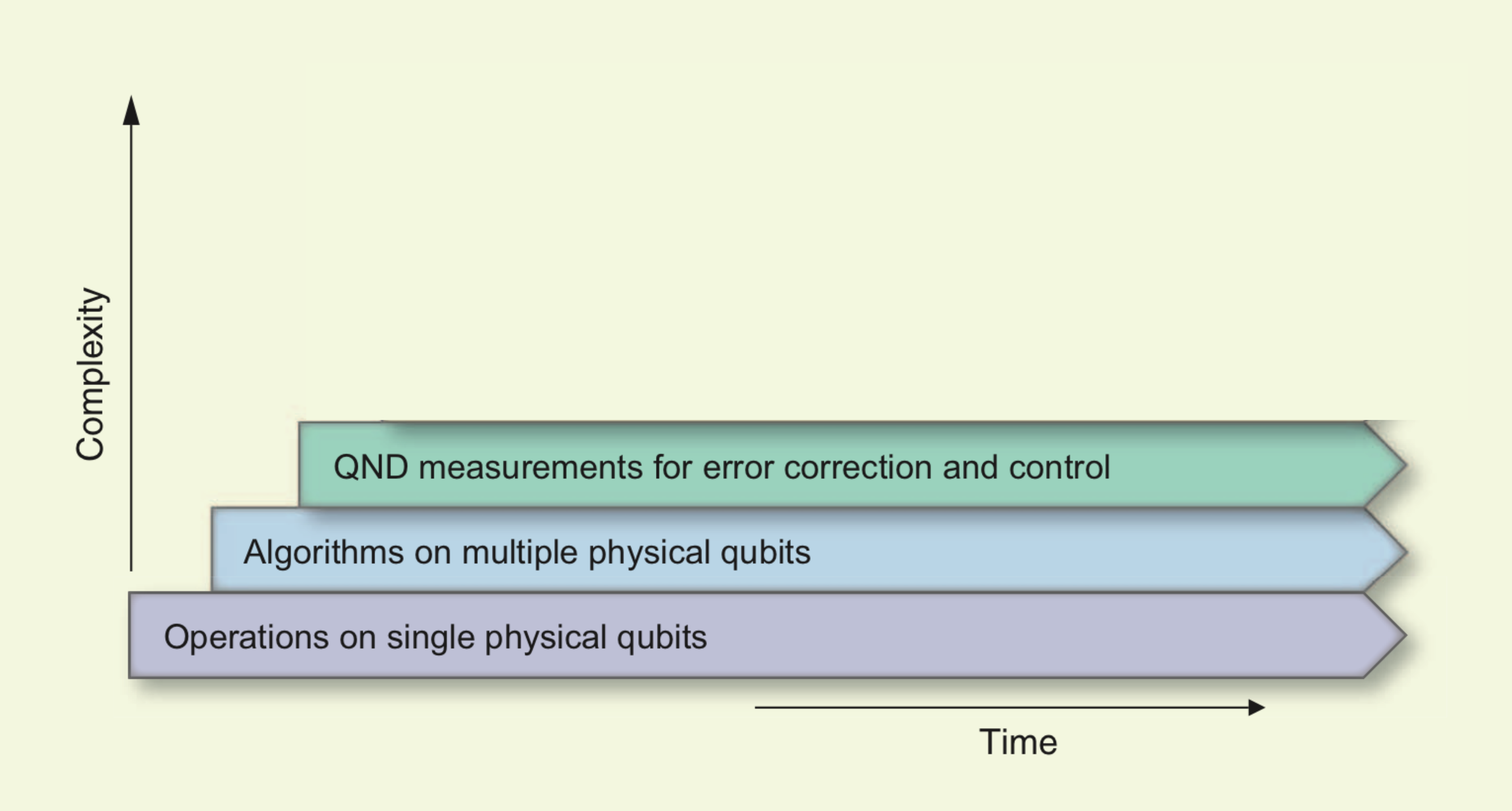
SHA-2

difficulty of factoring

difficulty of elliptic curve discrete logarithms

Can be solved efficiently by a large-scale quantum computer

Building quantum computers



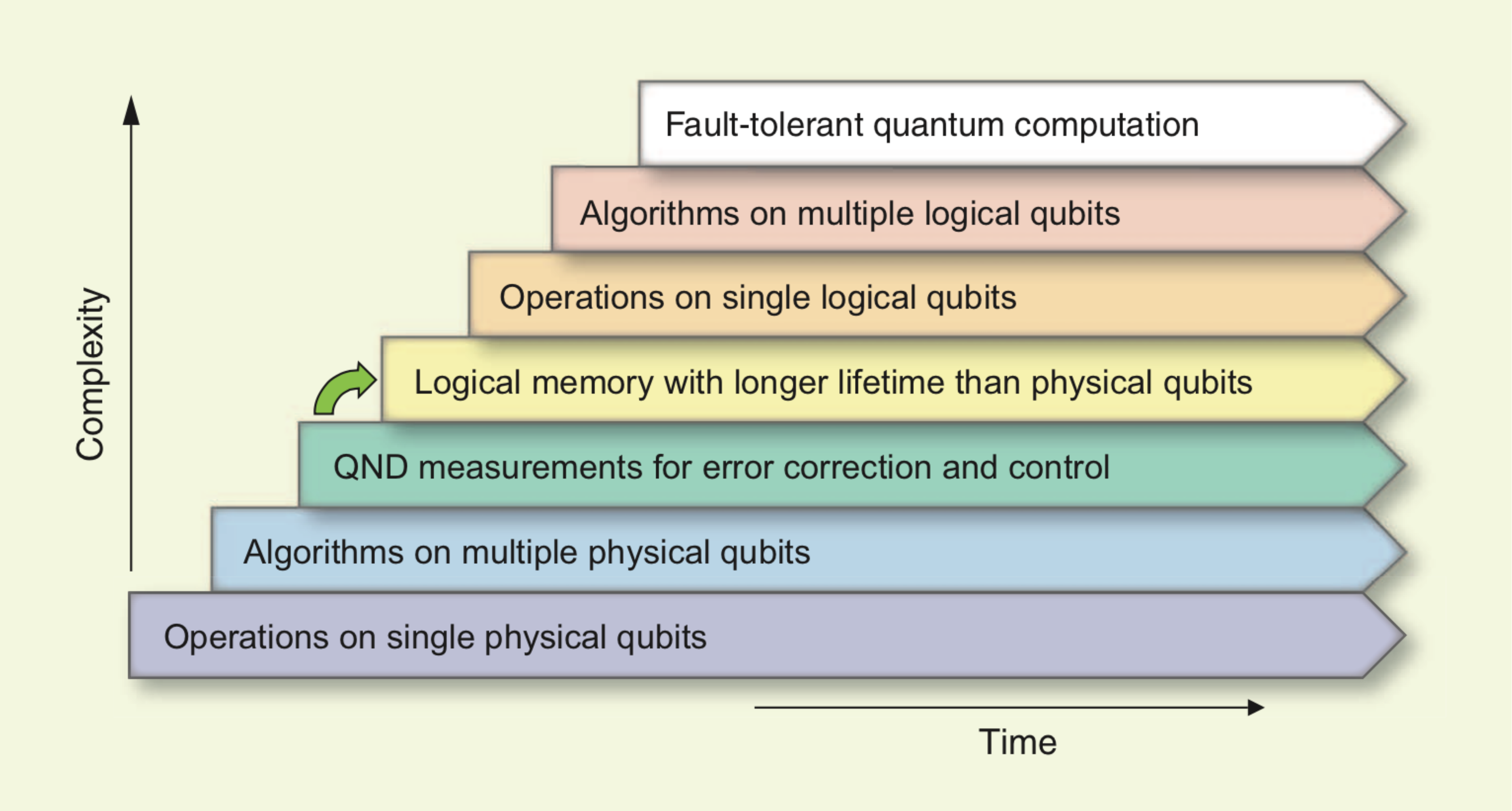
QND measurements for error correction and control

Algorithms on multiple physical qubits

Operations on single physical qubits

Devoret, Schoelkopf. *Science* 339:1169–1174, March 2013.

Building quantum computers



Devoret, Schoelkopf. *Science* 339:1169–1174, March 2013.

Post-quantum / quantum-safe crypto

No known exponential quantum speedup:

Code-based

- McEliece

Hash-based

- Merkle signatures
- Sphincs

Multivariate

- multivariate quadratic

Lattice-based

- NTRU
- learning with errors
- ring-LWE

Lots of questions

- Better classical or quantum attacks on post-quantum schemes?
- What are the right parameter sizes?
- Are the key sizes sufficiently small?
- Can we do the operations sufficiently fast?
- How do we integrate them into the existing infrastructure?

Lots of questions

This talk: ring learning with errors

- Are the key sizes sufficiently small?
- Can we do the operations sufficiently fast?
- How do we integrate them into the existing infrastructure?

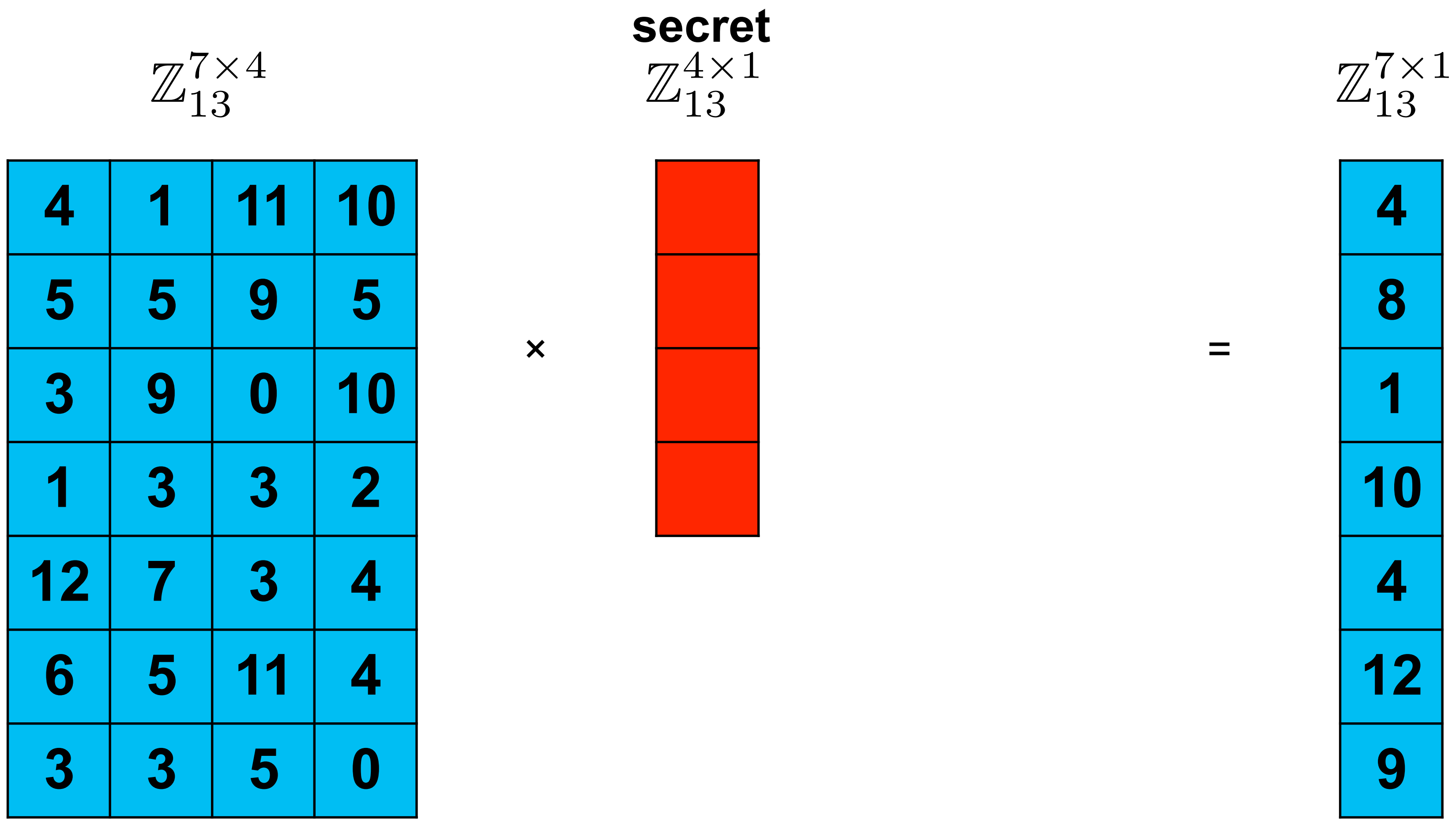
This talk: ring-LWE key agreement in TLS

Premise: large-scale quantum computers don't exist right now, but we want to protect today's communications against tomorrow's adversary.

- Signatures still done with traditional primitives (RSA/ECDSA)
  - we only need authentication to be secure *now*
  - benefit: use existing RSA-based PKI
- Key agreement done with ring-LWE

Learning with errors

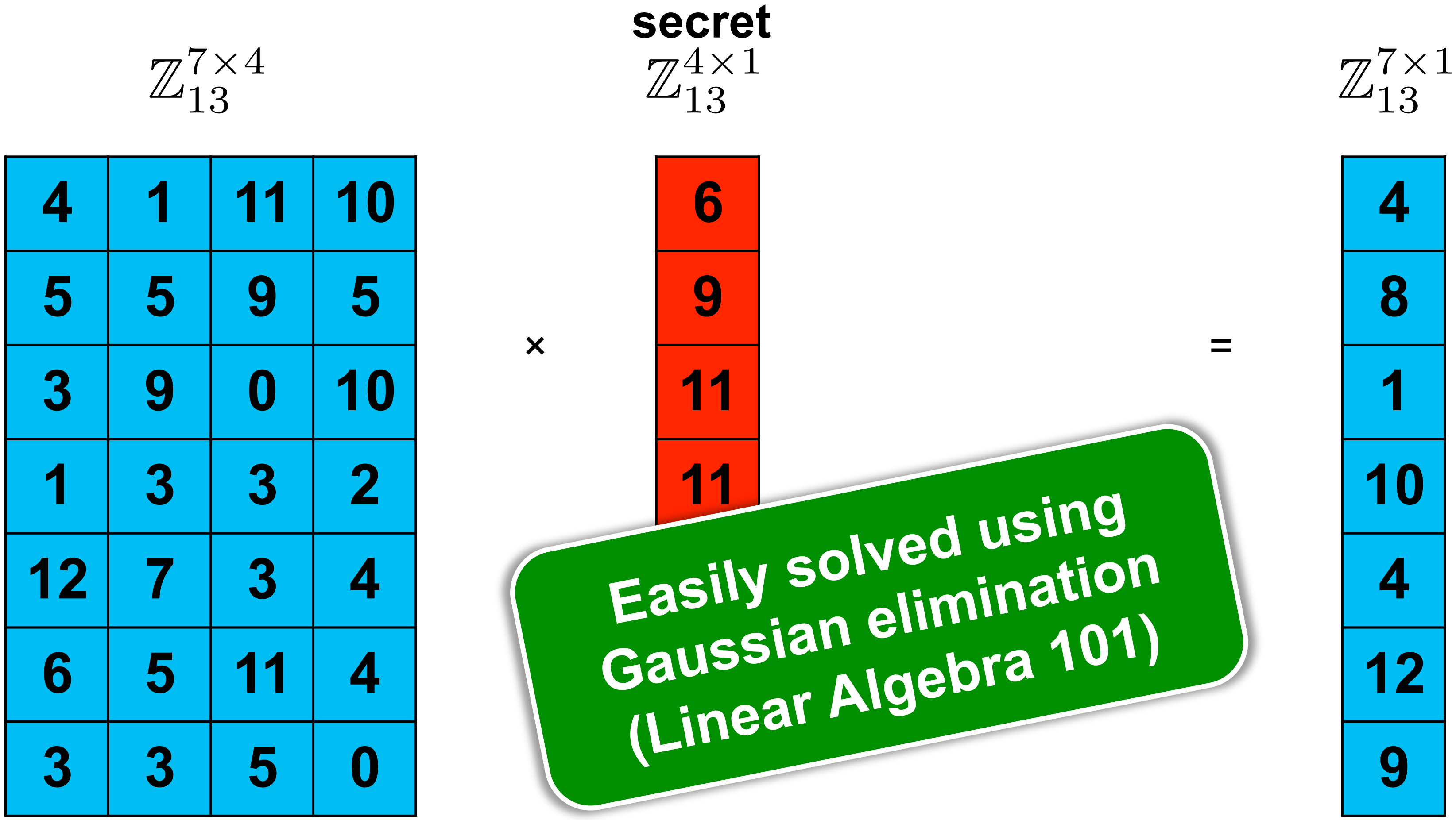
Solving systems of linear equations



$$\overset{\mathbb{Z}_{13}^{7\times 4}}{\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix}} \times \overset{\substack{\textbf{secret} \\ \mathbb{Z}_{13}^{4\times 1}}}{\begin{bmatrix} \ \\ \ \\ \ \\ \ \end{bmatrix}} = \overset{\mathbb{Z}_{13}^{7\times 1}}{\begin{bmatrix} 4 \\ 8 \\ 1 \\ 10 \\ 4 \\ 12 \\ 9 \end{bmatrix}}$$

Linear system problem: given **blue**, find **red**

Solving systems of linear equations



$\mathbb{Z}_{13}^{7\times 4}$

secret
$\mathbb{Z}_{13}^{4\times 1}$

$\mathbb{Z}_{13}^{7\times 1}$

$$\begin{bmatrix}4&1&11&10\\5&5&9&5\\3&9&0&10\\1&3&3&2\\12&7&3&4\\6&5&11&4\\3&3&5&0\end{bmatrix} \times \begin{bmatrix}6\\9\\11\\11\end{bmatrix} = \begin{bmatrix}4\\8\\1\\10\\4\\12\\9\end{bmatrix}$$

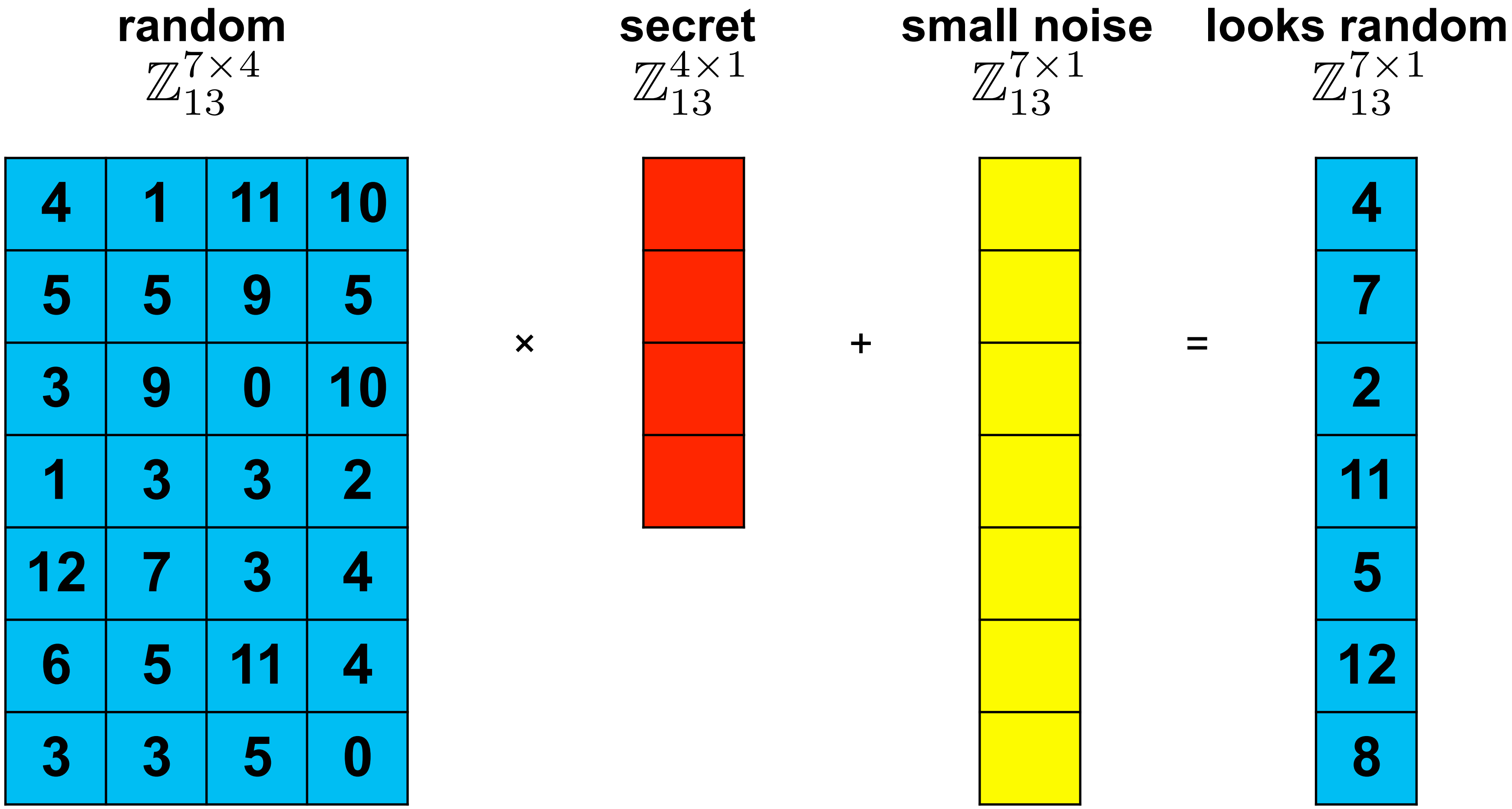
Easily solved using Gaussian elimination (Linear Algebra 101)

Linear system problem: given **blue**, find **red**

Learning with errors problem

$$\begin{array}{ccccccc}
\textbf{random} & & \textbf{secret} & & \textbf{small noise} & & \textbf{looks random} \\
\mathbb{Z}_{13}^{7\times 4} & & \mathbb{Z}_{13}^{4\times 1} & & \mathbb{Z}_{13}^{7\times 1} & & \mathbb{Z}_{13}^{7\times 1} \\
\begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} & + & \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} & = & \begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}
\end{array}$$

Learning with errors problem



random $\mathbb{Z}_{13}^{7\times 4}$

secret $\mathbb{Z}_{13}^{4\times 1}$

small noise $\mathbb{Z}_{13}^{7\times 1}$

looks random $\mathbb{Z}_{13}^{7\times 1}$

$$\begin{pmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{pmatrix} \times \begin{pmatrix} \\ \\ \\ \end{pmatrix} + \begin{pmatrix} \\ \\ \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{pmatrix}$$

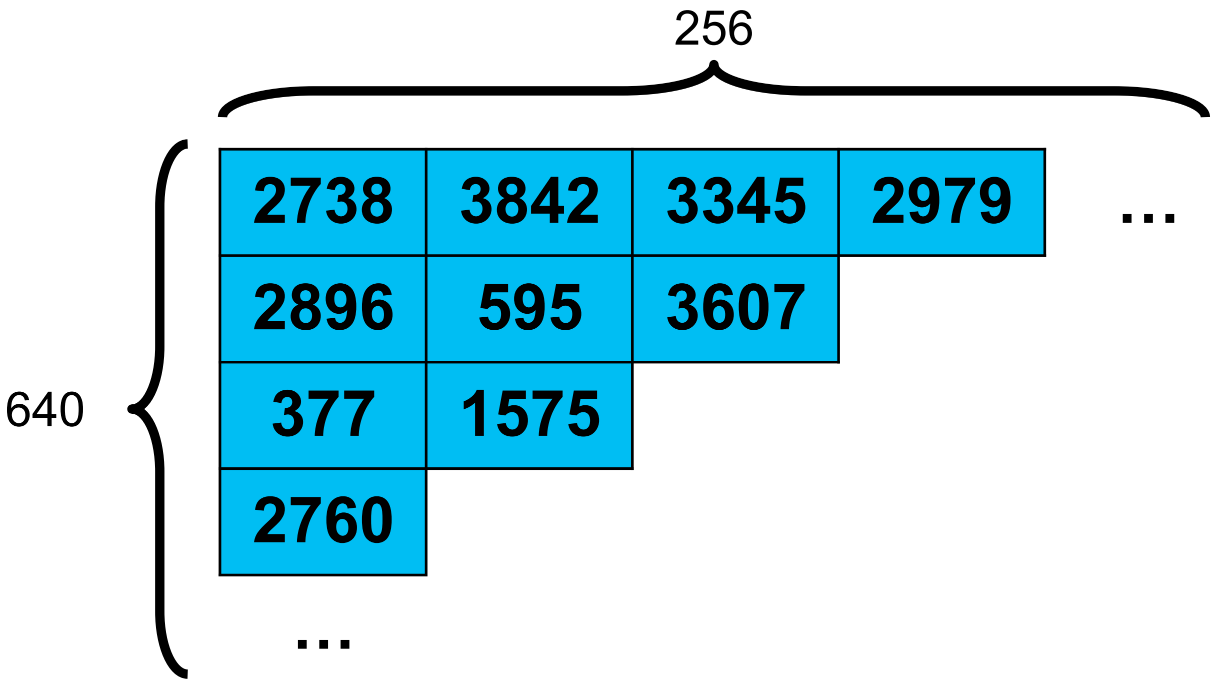
LWE problem: given **blue**, find **red**

Toy example versus real-world example

$\mathbb{Z}_{13}^{7\times 4}$

4	1	11	10
5	5	9	5
3	9	0	10
1	3	3	2
12	7	3	4
6	5	11	4
3	3	5	0

$\mathbb{Z}_{4093}^{640\times 256}$



2738	3842	3345	2979	…
2896	595	3607		
377	1575			
2760				
…				

640 × 256 × 12 bits = **245 KiB**

Ring learning with errors problem

random

$\mathbb{Z}_{13}^{7\times 4}$

4	1	11	10
10	4	1	11
11	10	4	1
1	11	10	4
4	1	11	10
10	4	1	11
11	10	4	1

Each row is the cyclic shift of the row above

Ring learning with errors problem

random
$\mathbb{Z}_{13}^{7\times 4}$

4	1	11	10
3	4	1	11
2	3	4	1
12	2	3	4
9	12	2	3
10	9	12	2
11	10	9	12

Each row is the cyclic shift of the row above

…

with a special wrapping rule: x wraps to $-x$ mod 13.

Ring learning with errors problem

random

$\mathbb{Z}_{13}^{7\times 4}$

4	**1**	**11**	**10**

Each row is the cyclic shift of the row above

…

with a special wrapping rule:
x wraps to –*x* mod 13.

So I only need to tell you the first row.

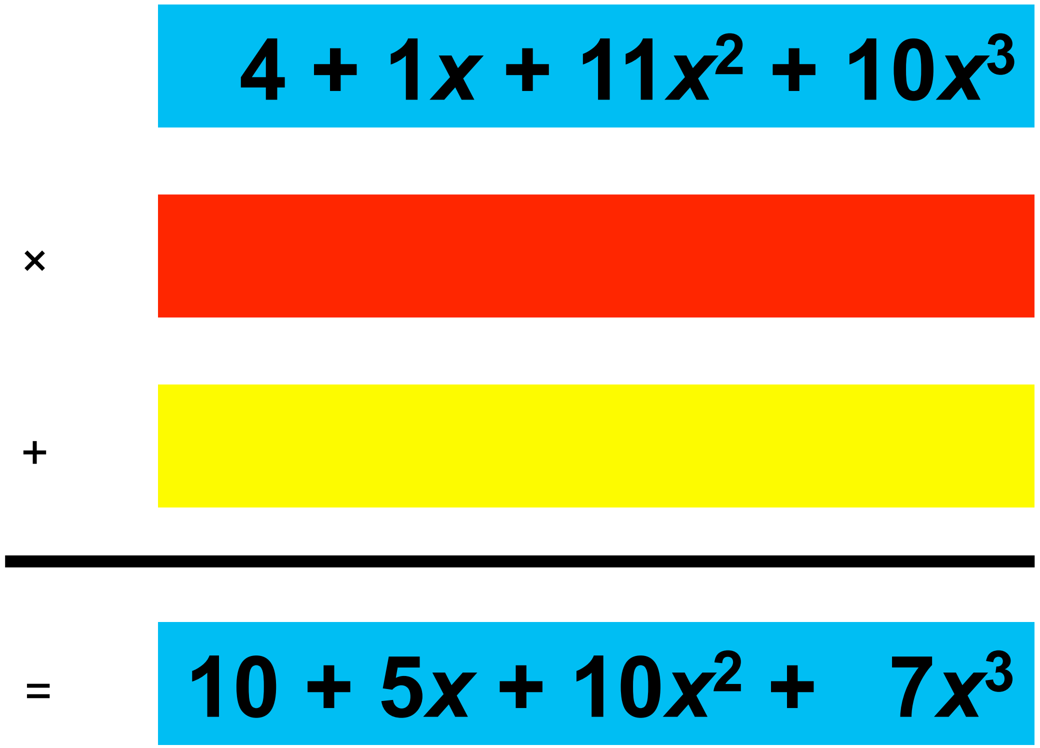
Ring learning with errors problem

$\mathbb{Z}_{13}[x]/\langle x^4 + 1\rangle$

	$4 + 1x + 11x^2 + 10x^3$	**random**
×	$6 + 9x + 11x^2 + 11x^3$	**secret**
+	$0 - 1x + 1x^2 + 1x^3$	**small noise**
=	$10 + 5x + 10x^2 + 7x^3$	

Ring learning with errors problem

$\mathbb{Z}_{13}[x]/\langle x^4 + 1\rangle$



$4 + 1x + 11x^2 + 10x^3$ **random**

× **secret**

\+ **small noise**

= $10 + 5x + 10x^2 + 7x^3$

Ring-LWE problem: given **blue**, find **red**

Decision ring learning with errors problem

$\mathbb{Z}_{13}[x]/\langle x^4 + 1\rangle$

	$4 + 1x + 11x^2 + 10x^3$	**random**
×	$6 + 9x + 11x^2 + 11x^3$	**secret**
+	$0 - 1x + 1x^2 + 1x^3$	**small noise**
=	$10 + 5x + 10x^2 + 7x^3$	**looks random**

Decision ring-LWE problem: given **blue**, distinguish **green** from random

Decision ring learning with errors problem with small secrets

$\mathbb{Z}_{13}[x]/\langle x^4 + 1 \rangle$

	$4 + 1x + 11x^2 + 10x^3$	**random**
×	$1 + 0x - 1x^2 + 2x^3$	**small secret**
+	$0 - 1x + 1x^2 + 1x^3$	**small noise**
=	$10 + 5x + 10x^2 + 7x^3$	**looks random**

Decision ring-LWE problem: given **blue**, distinguish **green** from random

Notation

- q: a prime
- n: a power of 2
- $R = \mathbb{Z}[X]/(X^n + 1)$: ring of polynomials in X with integer coefficients, polynomial reduction modulo $X^n + 1$
- $\mathbb{Z}_q$: integers modulo a prime q
- $R_q = \mathbb{Z}_q[X]/(X^n + 1)$: ring of polynomials in X with integer coefficients modulo q, polynomial reduction modulo $X^n + 1$

Decision ring learning with errors problem

Definition. Let n, R, q and R_q be as above. Let χ be a distribution over R, and let $s \overset{\$}{\leftarrow} \chi$. Define $O_{\chi,s}$ as the oracle which does the following:

1. Sample $a \overset{\$}{\leftarrow} \mathcal{U}(R_q)$, $e \overset{\$}{\leftarrow} \chi$,
2. Return $(a, as + e) \in R_q \times R_q$.

The *decision R-LWE problem* for n, q, χ is to distinguish $O_{\chi,s}$ from an oracle that returns uniform random samples from $R_q \times R_q$. In particular, if $\mathcal{A}$ is an algorithm, define the advantage

$$\mathrm{Adv}^{\mathsf{drlwe}}_{n,q,\chi}(\mathcal{A}) = \left| \Pr\left(s \overset{\$}{\leftarrow} \chi; \mathcal{A}^{O_{\chi,s}}(\cdot) = 1 \right) - \Pr\left(\mathcal{A}^{\mathcal{U}(R_q \times R_q)}(\cdot) = 1 \right) \right| \quad .$$

Hardness of DRLWE

Theory:

- There is a poly-time reduction from solving approximate shortest-independent vector problem (SIVP) on ideal lattices in *R* to solving DRLWE. [LPR10]

Practice:

- Assume the best way to solve DRLWE is to solve LWE.
- Solving LWE generally involves a lattice reduction problem.
- Albrecht et al. (eprint 2015/046) have hardness estimates.
- To get 160-bit classical security (≥ 80-bit quantum security):
  $n = 1024$, $q = 2^{32}-1$, chi = discrete Gaussian with parameter sigma = 8/sqrt(2π)

Key agreement

Basic ring-LWE-DH key agreement (unauthenticated)

- Reformulation of Peikert's R-LWE KEM (*PQCrypto 2014*)

public: "big" a in $R_q = \mathbf{Z}_q[x]/(x^n+1)$

Alice		**Bob**
secret: random "small" s, e in R_q		secret: random "small" s', e' in R_q
	$\xrightarrow{b = a \bullet s + e}$	
	$\xleftarrow{b' = a \bullet s' + e'}$	
shared secret: $s \bullet b' = s \bullet (a \bullet s' \bullet e') \approx s \bullet a \bullet s'$		shared secret: $b \bullet s' \approx s \bullet a \bullet s'$

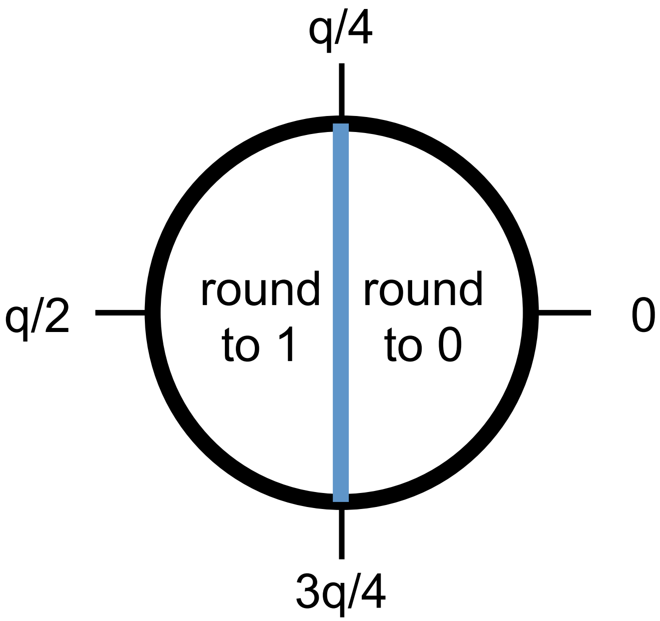
These are only approximately equal => need rounding

Rounding

- Each coefficient of the polynomial is an integer modulo q
- Treat each coefficient independently

Basic rounding

- Round either to 0 or q/2
- Treat q/2 as 1

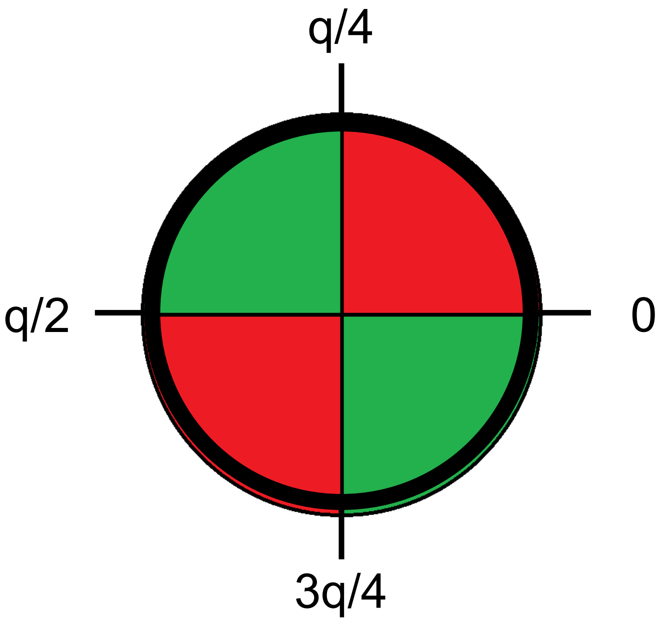


This works most of the time: prob. failure $1/2^{10}$.

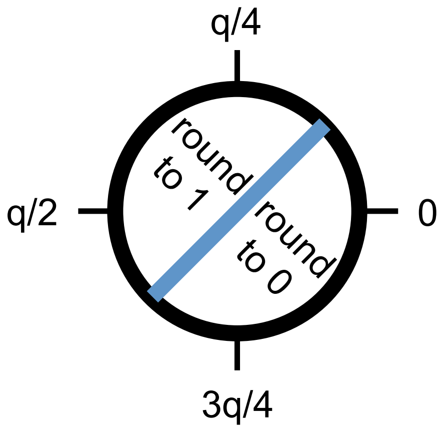
Not good enough: we need exact key agreement.

Better rounding (Peikert)

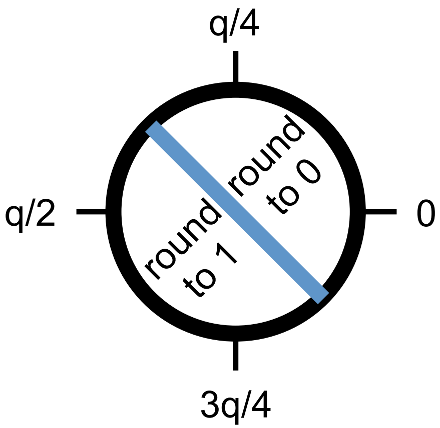
- Bob says which of two regions the value is in: 🟩 or 🟥



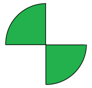
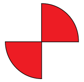
If 🟩



If 🟥



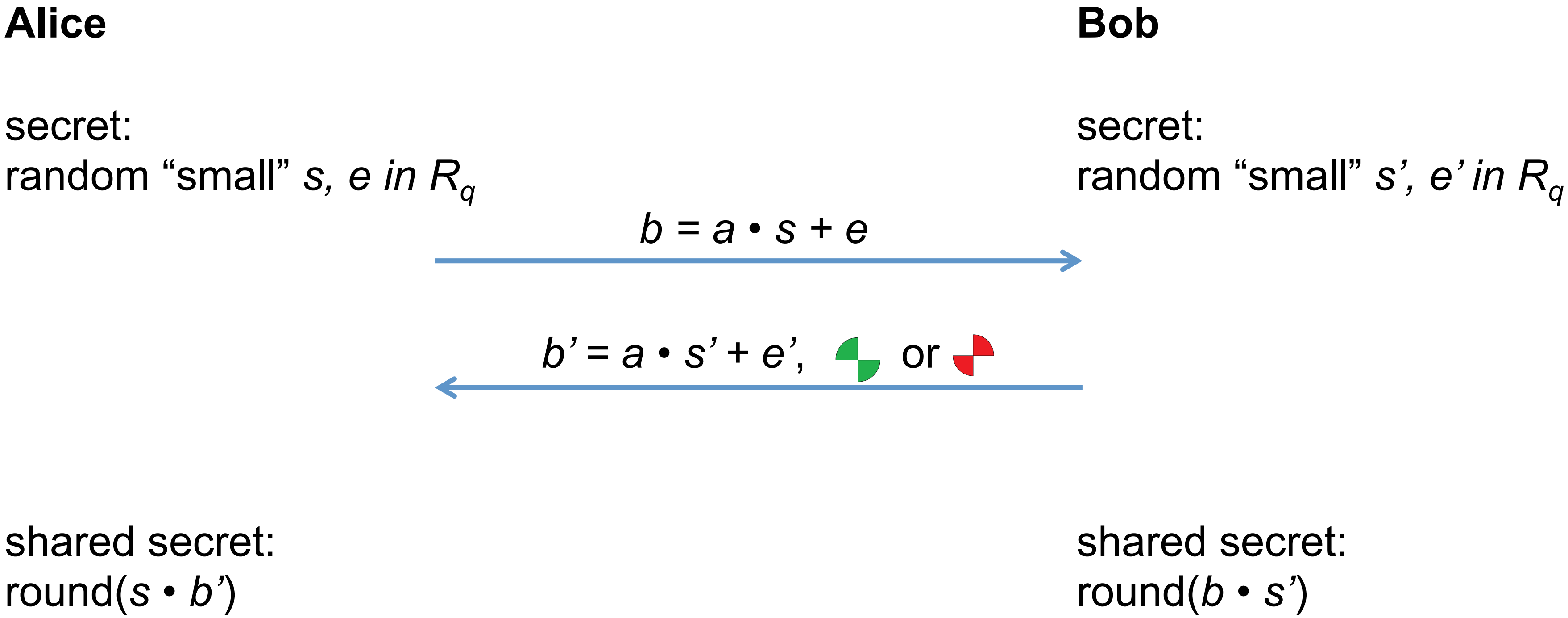
Better rounding (Peikert)

- If $|u–v| \leq q/8$, then this always works.
- For our parameters, probability $|u-v| > q/8$ is less than $2^{-128000.}$
- Security not affected: revealing  or  leaks no information

Exact ring-LWE-DH key agreement (unauthenticated)

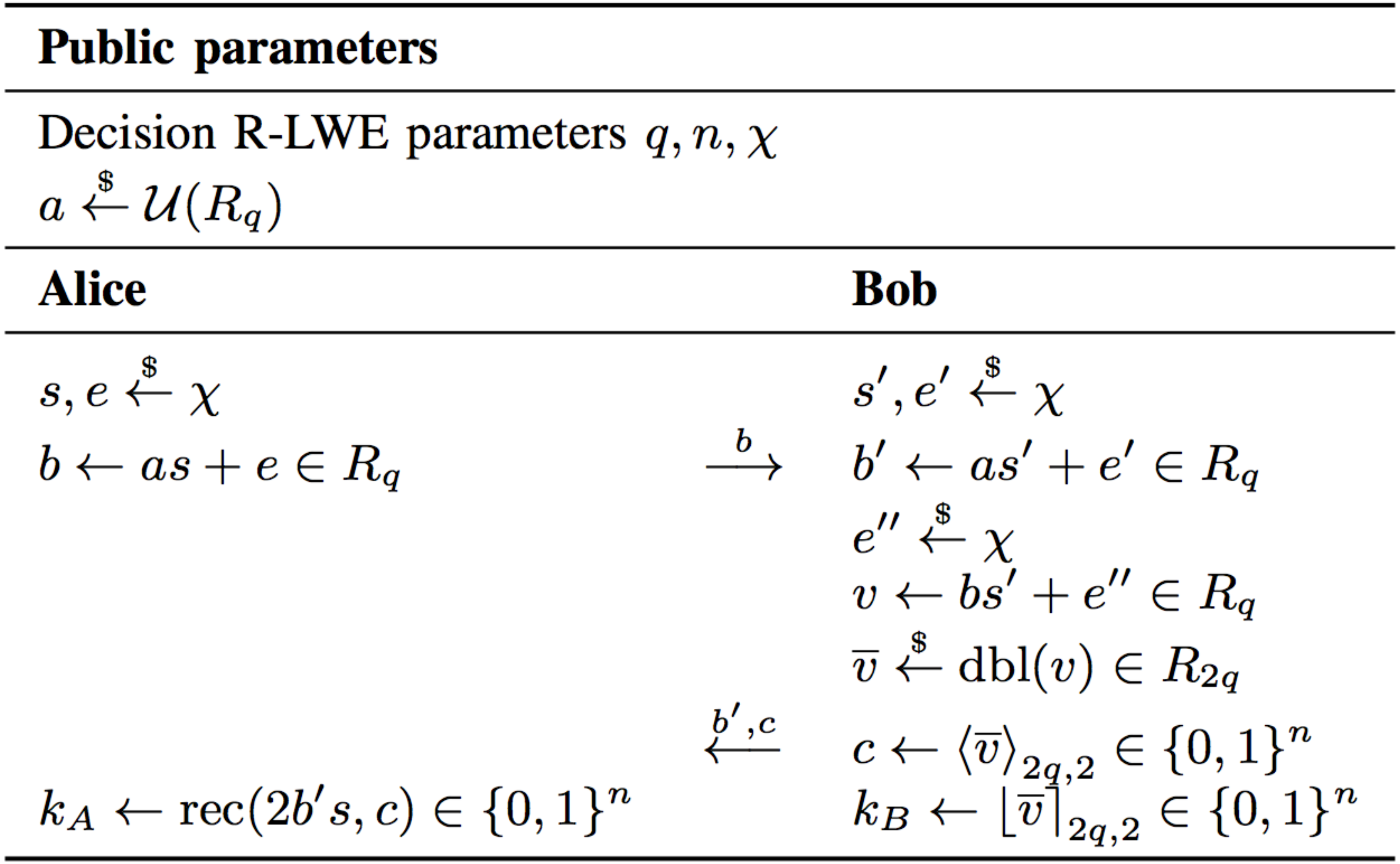
- Reformulation of Peikert's R-LWE KEM (*PQCrypto 2014*)

public: "big" a in $R_q = \mathbf{Z}_q[x]/(x^n+1)$



Alice		**Bob**
secret: random "small" s, e in R_q		secret: random "small" s', e' in R_q
	$b = a \bullet s + e$ →	
	← $b' = a \bullet s' + e'$, 🟩 or 🟥	
shared secret: round($s \bullet b'$)		shared secret: round($b \bullet s'$)

Ring-LWE-DH key agreement



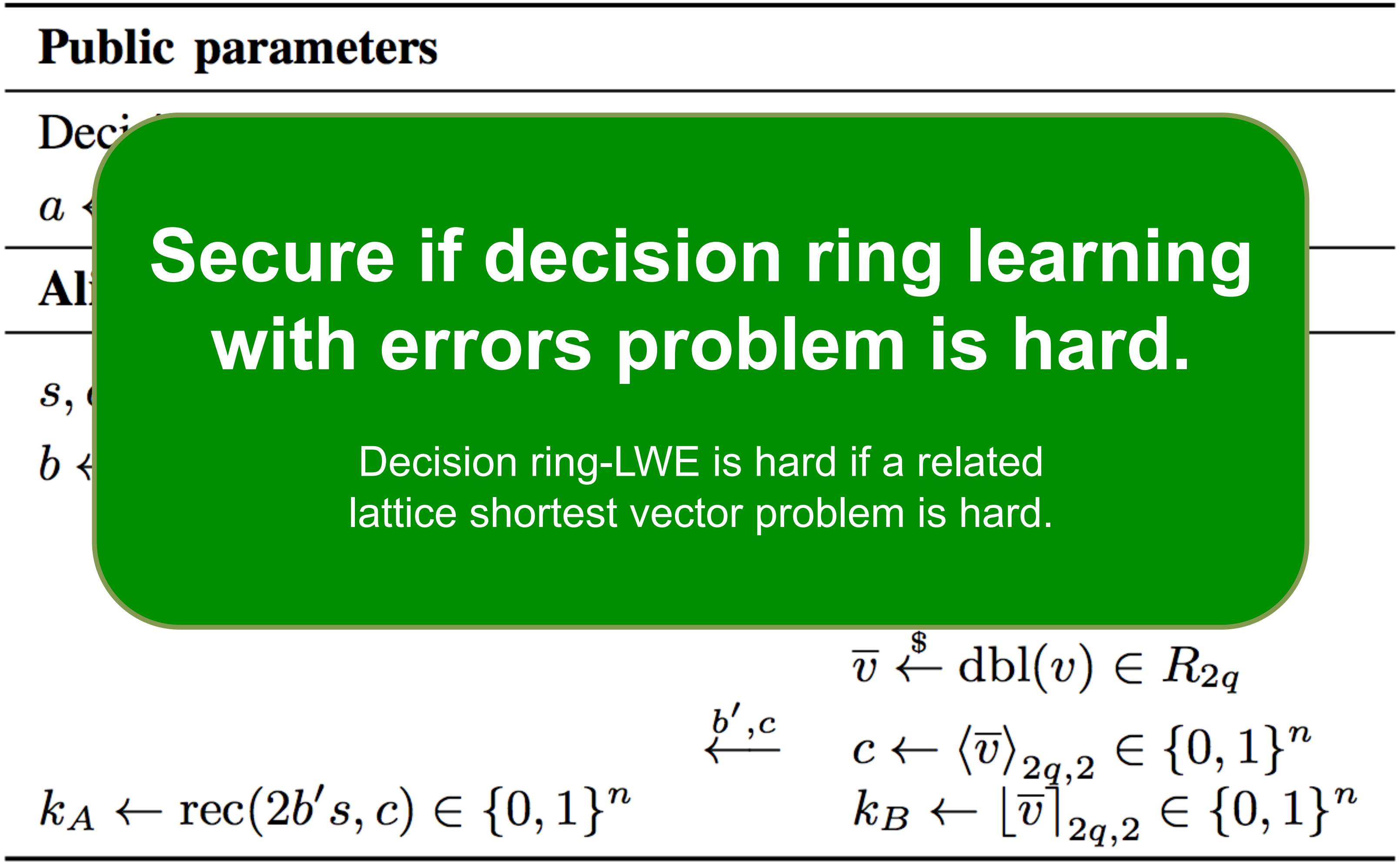
Public parameters

Decision R-LWE parameters q, n, χ

$a \xleftarrow{\$} \mathcal{U}(R_q)$

Alice		**Bob**
$s, e \xleftarrow{\$} \chi$		$s', e' \xleftarrow{\$} \chi$
$b \leftarrow as + e \in R_q$	$\xrightarrow{b}$	$b' \leftarrow as' + e' \in R_q$
		$e'' \xleftarrow{\$} \chi$
		$v \leftarrow bs' + e'' \in R_q$
		$\overline{v} \xleftarrow{\$} \mathrm{dbl}(v) \in R_{2q}$
	$\xleftarrow{b', c}$	$c \leftarrow \langle \overline{v} \rangle_{2q,2} \in \{0,1\}^n$
$k_A \leftarrow \mathrm{rec}(2b's, c) \in \{0,1\}^n$		$k_B \leftarrow \lfloor \overline{v} \rceil_{2q,2} \in \{0,1\}^n$

Ring-LWE-DH key agreement



Public parameters

Deci

a

Al

$s,$

b

$\overline{v} \xleftarrow{\$} \mathrm{dbl}(v) \in R_{2q}$

$\xleftarrow{b',c}$ $c \leftarrow \langle \overline{v} \rangle_{2q,2} \in \{0,1\}^n$

$k_A \leftarrow \mathrm{rec}(2b's, c) \in \{0,1\}^n$ $k_B \leftarrow \lfloor \overline{v} \rceil_{2q,2} \in \{0,1\}^n$

Secure if decision ring learning with errors problem is hard.

Decision ring-LWE is hard if a related lattice shortest vector problem is hard.

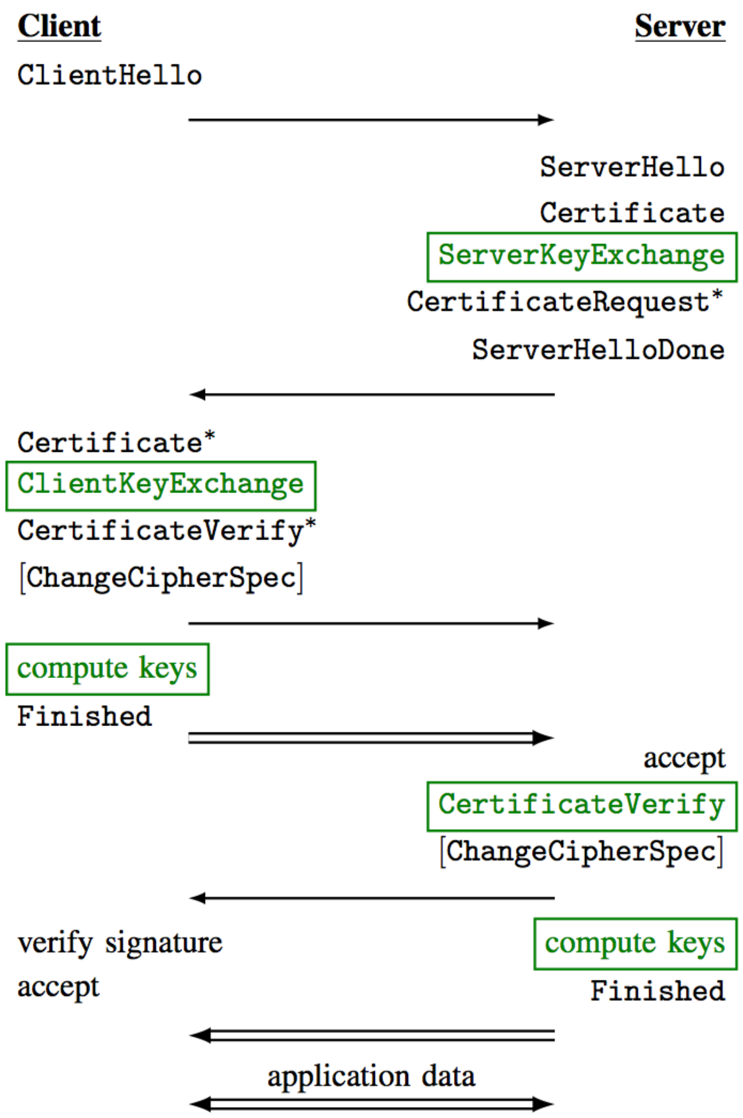
Implementation in TLS

Integration into TLS 1.2

New ciphersuite:

`TLS-RLWE-SIG-AES128-GCM-SHA256`

- RSA / ECDSA signatures for authentication
- Ring-LWE-DH for key exchange
- AES for authenticated encryption



Client

Server

ClientHello

ServerHello
Certificate
ServerKeyExchange
CertificateRequest*
ServerHelloDone

Certificate*
ClientKeyExchange
CertificateVerify*
[ChangeCipherSpec]

compute keys
Finished

accept
CertificateVerify
[ChangeCipherSpec]

verify signature
accept

compute keys
Finished

application data

Security within TLS 1.2

Model:

- authenticated and confidential channel establishment (ACCE) (Jager et al., *Crypto 2012*)

Theorem:

- signed ring-LWE ciphersuite is ACCE-secure if underlying primitives (signatures, ring-LWE, authenticated encryption) are secure
  - Interesting technical detail for ACCE provable security people: need to move server's signature to end of TLS handshake because oracle-DH assumptions don't hold for ring-LWE

Implementation

- Basic RLWE implemented in standalone C
  - two implementations: constant-time and non-constant-time
- Wrapped RLWE key exchange into OpenSSL libcrypto
- Added ciphersuites in OpenSSL libssl

Implementation aspect 1:

Polynomial arithmetic

- Polynomial multiplication in $R_q = \mathbf{Z}_q[x]/(x^{1024}+1)$ done with Nussbaumer's FFT:

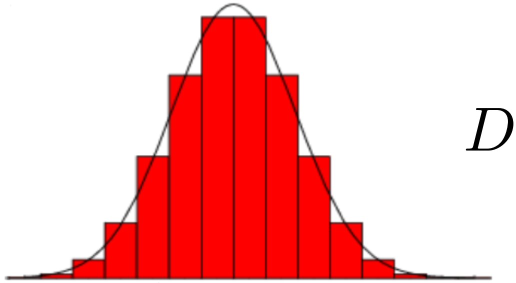
If $2^m = rk$, then

$$\frac{R[X]}{\langle X^{2^m}+1\rangle} \cong \frac{\left(\frac{R[Z]}{\langle Z^r+1\rangle}\right)[X]}{\langle X^k - Z\rangle}$$

- Rather than working modulo degree-1024 polynomial with coefficients in $\mathbf{Z}_q$, work modulo:
  - degree-256 polynomial whose coefficients are themselves polynomials modulo a degree-4 polynomial,
  - or degree-32 polynomials whose coefficients are polynomials modulo degree-8 polynomials whose coefficients are polynomials
  - or …

Implementation aspect 2:

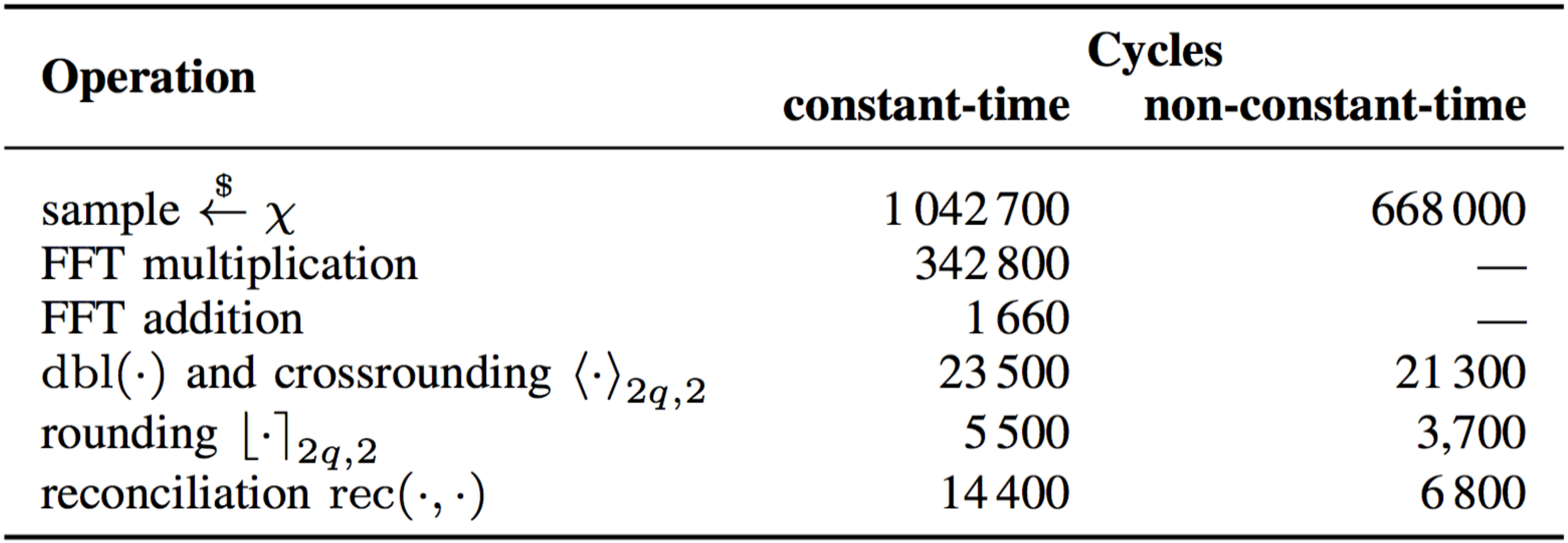
Sampling discrete Gaussians



$$D_{\mathbb{Z},\sigma}(x) = \frac{1}{S} e^{-\frac{x^2}{2\sigma^2}} \qquad \text{for } x \in \mathbb{Z}, \sigma \approx 3.2, S = 8$$

- Security proofs require “small” elements sampled within statistical distance 2^{-128} of the true discrete Gaussian
- We use inversion sampling: precompute table of cumulative probabilities
  - For us: 52 elements, size = 10000 bits
- Sampling each coefficient requires six 192-bit integer comparisons and there are 1024 coefficients
  - 51 • 1024 for constant time

Performance – math operations



Operation	**Cycles**	
	constant-time	**non-constant-time**
sample $\xleftarrow{\$} \chi$	1 042 700	668 000
FFT multiplication	342 800	—
FFT addition	1 660	—
$\mathrm{dbl}(\cdot)$ and crossrounding $\langle\cdot\rangle_{2q,2}$	23 500	21 300
rounding $\lfloor\cdot\rceil_{2q,2}$	5 500	3,700
reconciliation $\mathrm{rec}(\cdot,\cdot)$	14 400	6 800

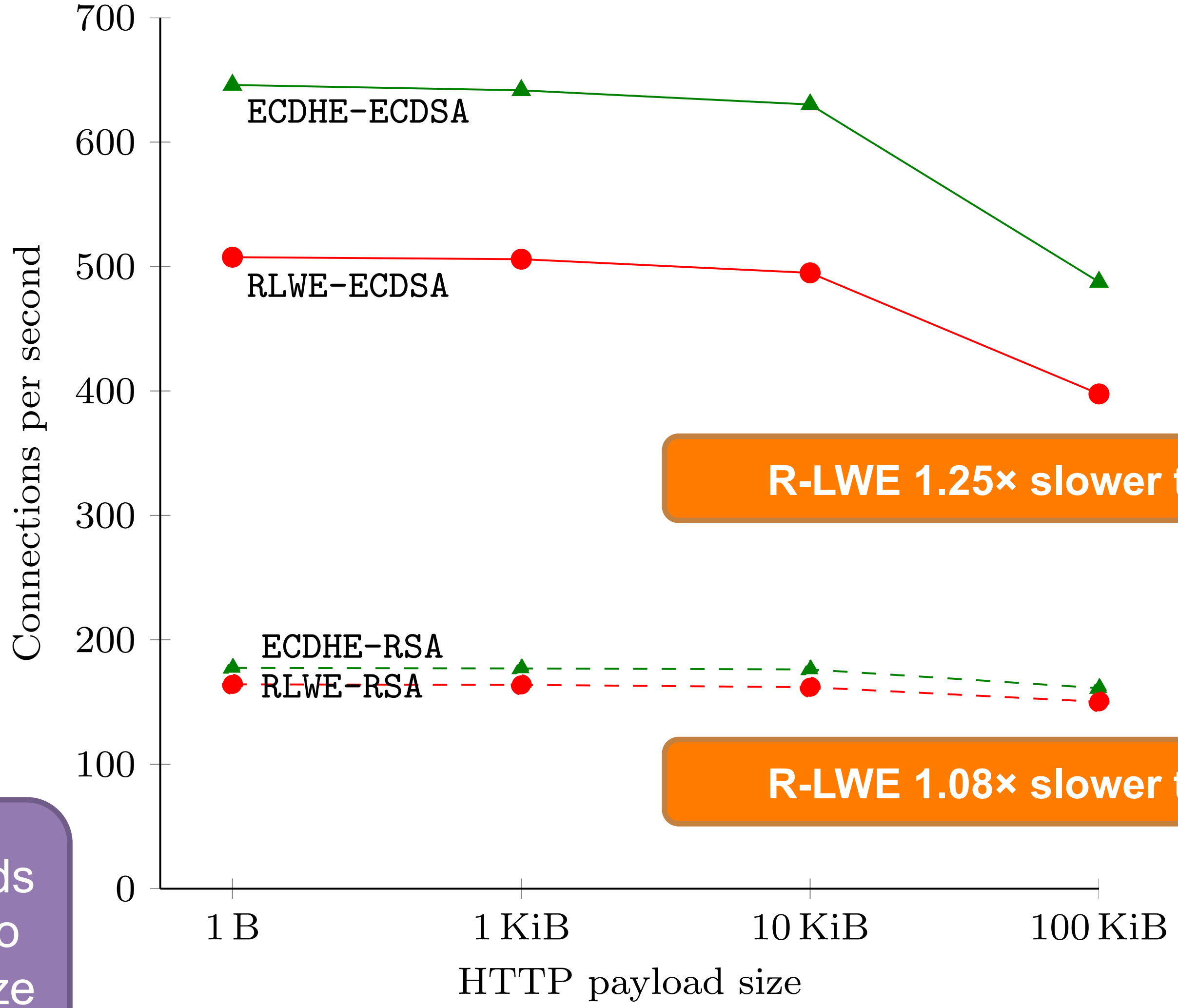
Performance – crypto operations

Operation	Client	Server
R-LWE key generation	0.9ms	0.9ms
R-LWE Alice	0.5ms	
R-LWE Bob		0.1ms
R-LWE total runtime	**1.4ms**	**1.0ms**
ECDH nistp256 (OpenSSL)	**0.8ms**	**0.8ms**

R-LWE 1.75× slower than ECDH

constant-time implementation
Intel Core i5 (4570R), 4 cores @ 2.7 GHz
llvm 5.1 (clang 503.0.30) –O3
OpenSSL 1.0.1f

Performance – in TLS

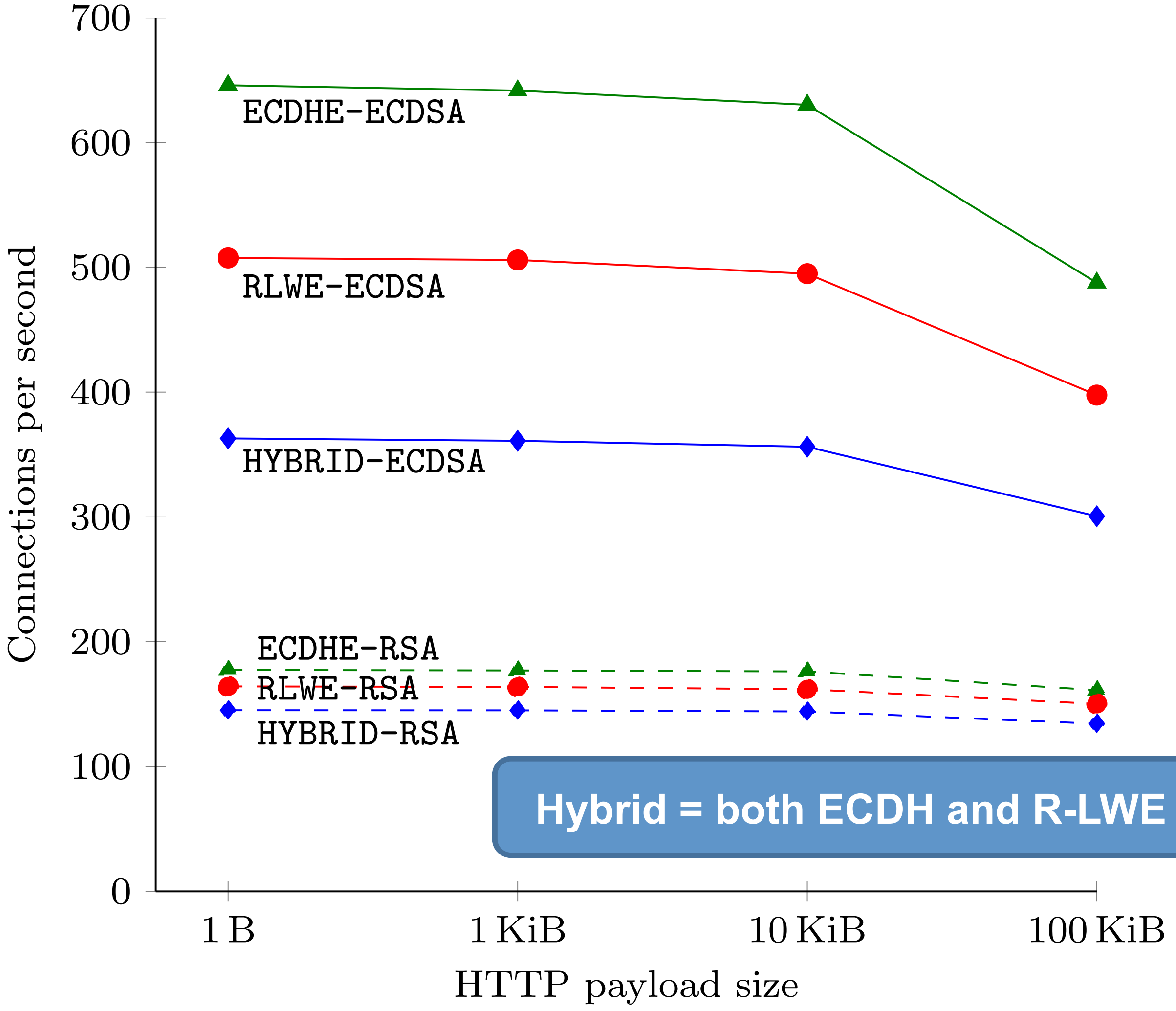


R-LWE 1.25× slower than ECDH

R-LWE 1.08× slower than ECDH

Ring-LWE adds about 8 KiB to handshake size

Performance – in TLS



Hybrid = both ECDH and R-LWE key exchange

Summary

Summary

Ring-LWE ciphersuite with traditional signatures:
- Key sizes: not too bad (8 KiB overhead)
- Performance: small overhead (1.1–1.25×) within TLS.
- Integration into TLS: requires reordering messages, but otherwise okay.

Caveat: lattice-based assumptions less studied, algorithms solving ring-LWE may improve, security parameter estimation may evolve.

Related / subsequent work

- Authenticated key exchange completely from RLWE
(Zhang, Zhang, Ding, Snook, Dagdalen, EUROCRYPT 2015)

- Hybrid RLWE + ECDH key exchange for Tor
(Ghosh, Kate, 2015)

- RLWE encryption on microcontrollers
(de Clercq, Roy, Vercauteren, Verbauwhede, 2015)

- NTRU-based key exchange for Tor
(Schanck, Whyte, Zhang, 2015)

Future work

better attacks / parameter estimation

- taking into account reduction tightness
- estimate based on best quantum algorithm for solving RLWE

ring-LWE performance improvements

- assembly
- alternative FFT
- better sampling, …

other post-quantum key exchange algorithms

- basic DH directly from LWE
- eCK-secure key exchange
- error correcting codes?

post-quantum authentication

Links

The paper

- http://eprint.iacr.org/2014/599

Magma code:

- http://research.microsoft.com/en-US/downloads/6bd592d7-cf8a-4445-b736-1fc39885dc6e/default.aspx

Standalone C implementation

- https://github.com/dstebila/rlwekex

Integration into OpenSSL

- https://github.com/dstebila/openssl-rlwekex